XXV. *The Properties of the mechanic Powers demonſtrated, with ſome Obſervations on the Methods that have been commonly uſed for that Purpoſe: in a Letter from* Hugh Hamilton, *D. D. F.R.S. and Fellow of* Trinity College, Dublin, *to* Matthew Raper, *Eſq; F. R. S.*

S I R, Trinity College, Dublin, 13 July, 1762.

Read April 21, and 28. 1763.

I Have here ventured to ſend you ſome remarks on the methods that have been commonly uſed in treating of thoſe engines that are called the mechanic powers: and alſo an account of the principles on which, I think, we may beſt explain their nature and manner of acting.

The many uſeful inſtruments that have been ſo ingeniouſly invented, and ſo ſuccesfully executed, and the great perfection to which the mechanic arts are now arrived, would naturally incline one to think that the true principles on which the efficacy and operations of the ſeveral machines depend, muſt long ſince have been accurately explained. But this is by no means a neceſſary inference: for, however men may differ in their opinions about the true method of accounting for the effects of the ſeveral machines, yet the practical principles of mechanics are ſo perfectly known by experience and obſervation, that the artiſt is thereby enabled to contrive and adjuſt the movements of of his engines with as much certainty and ſucceſs as he could do, was he thoroughly acquainted with the laws of motion, from which theſe principles may be ultimately derived. However, tho' an enquiry into the true method of deducing the practical principles of mechanics from the laws of motion, ſhould not perhaps contribute much to promote the progreſs of the mechanic arts, yet it is an enquiry in itſelf uſeful, and in ſome meaſure neceſſary: for, ſince late authors have uſed very different methods of treating this ſubject, it may be ſuppoſed that no one method has been looked upon as ſatisfactory and unexceptionable. I ſhould therefore wiſh to contribute towards having this ſubject treated with more accuracy than has been hitherto done.

The moſt general and remarkable theorem in machanics certainly is this, " That when two weights, by " means of a machine counterpoiſe each other, and " are then made to move together, their quantities of " motion will be equal". Now an æquilibrium always accompanying this equality of motions, bears ſuch a reſemblance to the caſe wherein two moving bodies ſtop each other, when they meet together with equal quantities of motion; that Doctor Wallis, and after him moſt of the late writers, have thought the cauſe of an æquilibrium in the ſeveral machines, might be immediately aſſigned: by ſaying, That, ſince one body cannot produce in another a quantity of motion equal to its own, without loſing its own at the ſame time; two heavy bodies, counteracting each other by means of a machine muſt continue at reſt, when they are ſo circumſtanced that one cannot deſcend, without cauſing the other to aſcend, at the ſame time; and with the ſame

ſame quantity of motion; and therefore two heavy bodies in ſuch caſes muſt always counterbalance each other. Now, this argument would be a juſt one, if it could properly be ſaid, that the motion of the aſcending body was produced by that of the deſcending one; but, ſince the bodies are ſo connected that one cannot poſſibly begin to move before the other, I apprehend, that, if the bodies are ſuppoſed to move, it cannot be ſaid that the motion of one is produced by that of the other: ſince whatever force is ſuppoſed to move one muſt be the immediate cauſe of motion in the other alſo; that is, both their motions muſt be ſimultaneous effects of the ſame cauſe, juſt as if the two bodies were really but one. And therefore if I was to ſuppoſe, in this caſe, that the ſuperior weight of the heavier body (which may be in itſelf much more than able to ſuſtain the lighter) ſhould overcome the weight of the lighter and produce equal motions in both bodies; I do not think that from thence I could be reduced to the abſurdity of ſuppoſing, that one body, by its motion, might produce in another, a motion equal to its own, and yet not loſe its own at the ſame time. But thoſe who argue from the equality of motions on this occaſion ſay further, that, ſince the two bodies muſt have equal motions when they do move, they muſt have equal endeavours to move even whilſt they are at reſt, and therefore theſe endeavours to move, being equal and contrary, muſt deſtroy each other, and the bodies muſt continue at reſt, and conſequently ballance each other. In anſwer to this I muſt obſerve, that the abſolute force with which a heavy body endeavours to deſcend from a ſtate of reſt can only be proportionable to its weight; and therefore I think

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it is neceſſary that ſome cauſe ſhould be aſſigned why (for inſtance) the endeavour of one pound to deſcend ſhall be equal to that of four pounds; and eſpecially as the fulcrum on which both weights act requires no greater force to ſupport it than that of five pounds.

From theſe conſiderations I infer, that the reaſon why very unequal weights may ballance each other, ſhould be aſſigned not from their having equal *momenta* when made to move together, but by proving *a priori* without conſidering their motions, that either the reaction of the fixed parts of the machine, or ſome other cauſe, ſo far takes off from the weight of the heavier body as to leave it only juſt able to ſupport the lighter. However, as this equality of *momenta* which always accompanies an *æquilibrium*, affords a very elegant theorem, it ought to be taken notice of in every treatiſe of mechanics, and may ſerve as an index of an *æquilibrium*. But I would not have it applied to a purpoſe for which it is unfit; as it has been in another inſtance by Doctor Keil, who from thence gives the reaſon why water ſtands at the ſame heighth in a narrow tube and a wide veſſel with which it communicates. And an argument of the ſame kind is applied ſtill more improperly by Dr. Rutherforth and others, to ſhew why a drop of water included in a ſmall conical tube will move towards the narrower end; and yet the true ways of accounting for both theſe phænomena are extremely obvious and eaſy.

The ſimple mechanic powers are uſually reckoned ſix, the lever, axle and wheel, pully, wedge, inclined plane, and ſcrew. The only method I have met with of explaining the nature of theſe machines on one principle, is that which I juſt now examined; and, as that appears to me unſatisfactory, I ſhall conſider the

the nature of each machine ſeparately in the order I have ſet them down.

The lever is ſaid to be a right line, inflexible and void of weight. Its fundamental property is this; when any two forces act againſt each other on the arms of a lever, they will continue in æquilibrio, if their quantities are inverſely as the diſtances between the points to which they are applied and the point round which the lever turns, which point is called the fulcrum or prop.

Several methods have been uſed, by different authors, to prove, that this property muſt neceſſarily belong to the lever. We find, in the works of Archimedes, a proof brought for this purpoſe, which has ſince been made uſe of by ſeveral writers of mechanics; who, I find, have ſomewhat altered the form of his argument, the ſubſtance of which is generally expreſſed as follows. — "When a cylinder of any uniform "matter is ſupported at its middle point, it will con- "tinue at reſt; for all the parts on one ſide muſt ba- "lance thoſe on the other, being exactly equal to "them both in weight and ſituation, ſo that the "whole weight of this cylinder may be looked up- "on as acting on the middle point on which it is ſup- "ported." From hence it is inferred, that the weight of ſuch a cylinder will act upon whatever ſupports it, in the ſame manner as it would do if it was all contracted into the middle point of its axis. If therefore we ſuppoſe the cylinder to be diſtinguiſhed into two unequal cylinders or ſegments, the diſtances between the middle points of thoſe ſegments and the middle of the whole cylinder will be inverſely as the lengths of the ſegments; that is, inverſely as their weights: but, as it was ſaid above, the weight of each cylinder acts in the ſame manner as it would do if contracted

contracted into the middle point of its axis; and therefore, if the weights of these cylinders be contracted into these points, they will continue to support each as before. And thence it is concluded, that any two weights, acting against each other on a line sustained at a fixed point, will counterpoise one another, when they are inversely as the distances of the points on which they act, from the point on which the line rests. To this argument there seems to be a manifest objection; for, when the whole cylinder is distinguished into two segments, part of the weight of the greater segment acts on the same side of the fulcrum with the lesser segment; and therefore when the whole weight of the greater segment is contracted into its middle point on one side of the fulcrum, and acts all of it against the lesser segment, it requires at least some proof to shew, that this contracted weight will be ballanced by the weight of the lesser segment. Mr. Hugens, in his Miscellaneous Observations on Mechanics takes notice of this objection to Archimedes's method, which, he says, several mathematicians had endeavoured to remove, but without success. He therefore, instead of this method, proposed one of his own, which depends on a postulatum that he uses in common with Archimedes, that I think ought not to be granted on this occasion; it is this: "When equal bodies are "placed on the arms of a lever, the one which is "furthest from the fulcrum will prevail and raise the "other up". Now this is taking it for granted, in other words, that a small weight placed further from the fulcrum will support or raise a greater one. The cause and reason of which fact must be derived from the demonstration that follows, and therefore this demonstration

ſtration ought not to be founded on the ſuppoſed ſelf-evidence of what is partly the thing to be proved. But perhaps it may be ſaid, that the poſtulatum may be granted merely on this account; the center of gravity of the two bodies (which in this caſe is the middle point between them) is not ſuſtained; and therefore the body which is on the ſame ſide of the fulcrum with the center of gravity will deſcend.

In anſwer to this I muſt obſerve, that this property, which the center of gravity has of deſcending, when not placed directly above or below the point of ſuſpenſion, cannot be proved to belong to it in any caſe, nor can we even ſhew that there is only one center of gravity between two bodies joined by a right line, until it is proved in general that the center of gravity of any two bodies is a point ſo placed between them that their diſtances from it are inverſely as their weights: but this in effect includes the principal property of the lever, which therefore cannot be proved from any previous ſuppoſition, that the center of gravity will deſcend, even when the bodies are equal, and we know it is the middle point between them.

I muſt now proceed to conſider what Sir Iſaac Newton hath delivered on this ſubject in his Principia, after the 2d cor. to the 3d law of motion which Dr. Clarke (in his notes on Rohault) and all the ſubſequent writers, have quoted as an elegant proof of the property of the lever; and therefore what appears to me at preſent an objection to this proof I ſhall mention with great diffidence, and in hopes of being ſet right if I am wrong. Sir Iſaac ſuppoſes two weights, as A and P. Tab. IV. Fig. 1. to hang by threads, from the points M and N, in a wheel or circular plane perpendicular to the horizon,

rizon and movable about its center O; and then proposes to determine the forces which these weights have to turn the wheel round its center. In order to do this, he supposes that it is indifferent from what points in the perpendicular lines M A and N P the weights are hung, for that they will still have the same power to turn the wheel about its center. His words are: "Quoniam nil refert utrum filorum puncta K, L, D, "affixa sint vel non affixa ad planum rotæ; pon-"dera idem valebunt ac si suspenderentur a punctis "K et L, vel D et L". Now whether the points of the threads K, L, D, are fixed or not to the plane of the wheel is certainly of importance, as it must make a difference in the points of suspension of the weights, and consequently in the degrees of obliquity with which the weights act; for the lowest point of the thread that is fixed to the plane must be considered as the point from which the weight hangs; as the parts of the thread above that point are quite useless not being at all acted upon. And from thence I shall endeavour to shew that to suppose the weight A will have the same power to turn the wheel from whatever point in the line M A it hangs, is in effect presupposing what is intended to be proved. For it appears, from what he says immediately after, that, when the weight A hangs from the point D, if its whole force be expressed by the line A D, and be resolved into two forces, D C and A C, the former only will have any effect in turning the wheel, as it acts perpendicularly on the radius O D, while the latter is lost, its direction being parallel to O D. But it is evident, that, when the same weight hangs from the point K, as it acts perpendicularly on the radius O K, its whole force is is exerted to turn the wheel, and none of it loſt by oblique action. Therefore the force which the weight A, exerts to oppoſe the weight P, and turn the wheel when it hangs from D, is, to the force it exerts when it hangs from K, as the line D C to A D, or as O K, to O D, (ſim. triang. A D C, D O K) that is the force exerted by the weight A, hanging from the points D, and K, are inverſely as the radii O D, and O K. And therefore to ſuppoſe, that theſe two forces will have the ſame effect in turning the wheel and oppoſing the weight P, is the ſame as ſuppoſing that two forces will have equal effects in moving the arms of a lever (on which they act perpendicularly) when they are inverſely as the lengths of thoſe arms. — But this is the very concluſion Sir Iſaac draws from his premiſes, for he ſays: "Pondera igitur A & P, "quæ ſunt reciproce ut radii in directum poſiti O K, "O L, idem pollebunt et ſic conſiſtent in æquili- "brio, quæ eſt proprietas notiſſima libræ vectis et "axis in peritrochio". This property of the lever, which is here expreſſed in general terms, includes two caſes. For the arms of the lever may be either perpendicular or oblique to the directions of the weights. The firſt of theſe caſes is the ſimpleſt, and ſhould be firſt demonſtrated: And I do not ſee how there can be any room for applying the reſolution of forces in demonſtrating this caſe, in which no part of either weight is loſt by oblique action. But when this caſe is proved, we have from thence, by the reſolution of forces, an eaſy way of ſhewing, in the ſecond caſe, when the arms of the lever are oblique to the directions of the weights, that the weights will counterballance each other, when they are reciprocally as the perpendicular diſtances of their

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lines of direction from the center of motion. — From either of these cases, we may deduce an obvious reason why the weight A, should have the same power to turn the wheel, from whatever point it hangs in the line M A; the truth of which, I am persuaded, cannot be proved independent of those cases, and therefore think it ought not to be used as a postulatum in demonstrating the general property of the lever.

Mr. Maclaurin, in his View of Newton's Philosophy, after giving us the methods which Archimedes and Newton have used for proving the fundamental property of the lever, proposes one of his own, which, he says, appears to be the most natural one for this purpose. However as to his method I shall only observe, that from equal bodies sustaining each other at equal distances from the fulcrum, he shews us how to infer that a body of one pound (for instance) will sustain another of two pounds at half its distance from the fulcrum, and from thence that it will sustain one of three pounds at a third of its distance from the fulcrum: and thus he goes on deducing, by a kind of induction, what the proportion is in general between two bodies that sustain each other on the arms of a lever. But this argument (which I do not think by any means satisfactory) he observes cannot be applied when the arms of the lever are incommensurable.

These are the methods of demonstrating the fundamental property of the lever, which are most worth taking notice of; and, since they seem liable to objections, and the other methods I have met with are still more exceptionable, I shall propose a new proof of this property of the lever, which appears to me

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a very ſimple one, and depends on a poſtulatum that, I believe, will be readily granted.

If a force be uniformly diffuſed over a right line, that is, if an equal part of the force acts upon every point of the line, and if the whole force acts according to one and the ſame plane; this force will be ſuſtained, and the line kept in æquilibrio, by a ſingle force applied to the middle point of the line equal to the diffuſed force, and acting in a contrary direction.

In order to ſhorten the following proof, I muſt premiſe by way of Lemma, that, if a right line be divided into two ſegments, the diſtances between the middle of the whole line, and the middle points of the ſegments, will be inverſely as the ſegments. This is ſelf evident when the ſegments are equal; and, when they are unequal, then, ſince half of the whole line is equal to half of the greater and half of the leſſer ſegment, it is plain that the diſtance between the middle of the whole line and the middle of one ſegment muſt be equal to half of the other ſegment, ſo that theſe diſtances muſt be to each other inverſely as the ſegments, all which appears evident from the inſpection of TAB. VI. Fig. 2.

Let now the line G H, whoſe middle point is D, be divided into the unequal ſegments G L, and L H, whoſe middle points are C and F, and let two forces or weights, A and B, which are to each other as the ſegments G L and L H, be applied to their middle points C and F, and let them act perpendicularly on the line G H. Then (by the Lemma) the weights A and B will be to each other inverſely as C D, and F D, (the diſtances of the points C and F, to which

they

they are applied from the middle of the whole line) if then a third force or weight E, equal to the ſum of the forces A and B, be applied to the point D, and acts on the line in an oppoſite direction; I ſay theſe three forces will ſuſtain each other, and keep the line in æquilibrio. For let us ſuppoſe the force E to be removed, and inſtead of it another force, equal alſo to the ſum of A and B, to be uniformly diffuſed over the whole line G H, and to act directly againſt the forces A and B, then the part of this force which acts on the ſegment G L, will be equal to the force A, and therefore will be ſuſtained by it (poſtulatum); and the other part, which is diffuſed over the ſegment L H, will be equal to and ſuſtained by the force B, ſo that the forces A and B will ſuſtain this diffuſed force and keep the line in æquilibrio. — Let now two other forces act on this line in oppoſite directions, one of them the force E acting on the point D, as it was firſt ſuppoſed to do, and the other an uniformly diffuſed force equal to E (and conſequently equal to the other diffuſed force), then theſe two additional forces will alſo ballance each other, and therefore the æquilibrium will ſtill remain. So that the two forces A and B, and a diffuſed force acting on one ſide of the line ſuſtain the force E, and a diffuſed force acting on the other ſide: but it is manifeſt, that, in this æquilibrium, the two diffuſed forces acting on oppoſite ſides are perfectly equivalent, and therefore, if they are taken away from both ſides, the æquilibrium muſt ſtill remain. Hence it appears that the three weights or forces A, B and E, any two of which are (by the conſtruction) to each other inverſely as their diſtances from the third, will ſuſtain each other, and keep the line on which they

they act in æquilibrio: which is the first and most simple case of the property of the lever; for here the directions of the weights are supposed to be perpendicular to the line on which they act, and it is evident that, if one of the points C, D or F, be fixed or considered as a fulcrum, that the weights acting on the other two points will continue to support each other. I shall not trouble you with proving the second case of the property of the lever; it is most easily deduced from the first: for, when two weights act on the arms of a lever in oblique directions, and are to each other inversely as the perpendicular distances of their lines of direction from the center of motion, then, by the resolution of forces, it is easily proved, that the parts of those forces which act perpendicularly on the arms of the lever, and which only are exerted to turn the lever, are to each other inversely as the lengths of those arms; and therefore by the first case they must ballance each other.

I shall now mention some well known truths in mechanics, which, I think, cannot be proved otherwise than by deducing them from what hath been here demonstrated.

COR. I.

It appears from hence, that the powers with which any two forces move or endeavour to move the arms of a lever, are as the rectangles, under lines proportional to the forces, and the perpendicular distances of their lines of direction from the fulcrum.

COR. II.

When therefore two bodies acting on the arms of a lever ſuſtain each other, if one of them be removed farther from the fulcrum, it will preponderate; but if it be brought nearer to the fulcrum, the other weight will prevail: becauſe the product to which its force is proportional will be encreaſed in the firſt caſe, and diminiſhed in the ſecond.

COR. III.

We learn from hence, to find out the center of gravity of any two bodies joined by an inflexible right line; and to prove that its definition will agree to one point only in the line. For if a point be taken in the line ſo that the diſtances of the bodies from it may be inverſely as their weights, that point will be their center of gravity, becauſe, when it is ſuſtained, the bodies will be in æquilibrio. But if the line be ſuſtained at any other point, then is the fulcrum removed farther from one body and brought nearer to the other than it was when the bodies ballanced each other; and therefore, by the preceding Cor. that body from which it is removed, or which is on the ſame ſide with the center of gravity, will deſcend. Conſequently there is but one point in the line, which being ſuſtained, the bodies will continue in æquilibrio, and therefore but one point only can be their center of gravity. Hence alſo it appears, that the center of gravity will always deſcend, when it is not directly above or below the point by which the body is ſuſtained.

I ſhall now endeavour to be as conciſe as poſſible in what I have to ſay of the other mechanic powers; having, I fear, been too tedious in my account of the lever, which however deſerves to be particularly conſidered, ſince to it may be reduced the ballance, the axle and wheel, and (according to ſome writers) the pulley.

The ballance I do not conſider as a diſtinct machine, becauſe it is evidently no other than a lever fitted to the particular purpoſe of comparing weights together, and does not ſerve for raiſing weights, or overcoming reſiſtances, as the other machines do.

When a weight is to be raiſed by means of an axle and wheel it is faſtened to a chord that goes round the axle, and the power which, is to raiſe it is hung to a chord that goes round the wheel. If then the power be to the weight as the radius of the axle to the radius of the wheel, it will juſt ſupport that weight; as will eaſily appear from what was proved of the lever. For the axle and wheel may be conſidered as a lever, whoſe fulcrum is a line paſſing through the center of the wheel and middle of the axle, and whoſe long and ſhort arms are the radii of the wheel and axle which are parallel to the horizon, and from whoſe extremities the chords hang perpendicularly. And thus an axle and wheel may be looked upon as a kind of perpetual lever, on whoſe arms the power and weight always act perpendicularly, tho' the lever turns round its fulcrum. And in like manner when wheels and axles move each other by means of teeth on their peripheries, ſuch a machine is, really, a perpetual compound lever: and, by conſidering it as ſuch, we may compute the proportion of any power to the weight it it is able to ſuſtain by the help of ſuch an engine. And ſince the radii of two contiguous wheels, whoſe teeth are applied to each other, are as the number of teeth in each, or inverſely as the number of revolutions, which they make in the ſame time; we may, in the computation, inſtead of the ratio of theſe radii, put the ratio of the number of teeth on each wheel; or the inverſe ratio of the number of revolutions they make in the ſame time.

Some writers have thought the nature and effects of the pulley might be beſt explained by conſidering a fixed pulley as a lever of the firſt, and a moveable pulley as one of the ſecond kind. But tho' the pulley may bear being conſidered in that light; yet, I think, the beſt and moſt natural method of explaining its effects (that is, of computing the proportion of any power to the weight it can ſuſtain by means of any ſyſtem of pulleys) is, by conſidering that every moveable pulley hangs by two ropes equally ſtretched, which muſt bear equal parts of the weight: and therefore when one and the ſame rope goes round ſeveral fixed and moveable pullies, ſince all it parts on each ſide of the pullies are equally ſtretched, the whole weight muſt be divided equally amongſt all the ropes by which the moveable pullies hang. And conſequently if the power which acts on one rope be equal to the weight divided by the number of ropes, or double the number of moveable pullies, that power muſt ſuſtain the weight.

Upon this principle, the proportion of the power to the weight it ſuſtains by means of any ſyſtem of the pullies, may be computed in a manner ſo eaſy and natural as muſt be obvious to every common capacity.

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The proportion, which any power bears to the refifting force it is able to fuftain by means of a wedge, has been laid down differently by different authors; fome of whom therefore muft have been miftaken: and none of them feem to have treated the matter fo generally as they might have done. Without examining their feveral opinions, I fhall proceed to confider what proportion a power acting on a wedge muft have to the refiftance it fuftains in three different cafes, to which I think all thofe relating to the wedge may be reduced.

Firft, When the refifting bodies act perpendicularly on the fides of the wedge, and recede alfo in lines perpendicular to the fides.

Secondly, When the refifting bodies act on the wedge in oblique directions equally inclined to the fides, and recede in lines perpendicular to the fides.

Thirdly, When the refifting bodies are confined, by planes put under them, or otherwife, to recede in particular directions oblique to the fides.

CASE I.

Let the æquicrural triangle A, B, C, [TAB. VI. Fig. 3.] reprefent a wedge on whofe fides the two equal refifting forces E and F act perpendicularly with lines of direction meeting at the point D, at which the power P acts perpendicularly on the bafe A C. Then, fince thefe three forces are fuppofed to fuftain each other and keep the wedge in æquilibrio, they muft be to each other as the fides of a triangle to which their directions are parallel, and therefore they will be to each other as the fides of a triangle to which their directions

directions are perpendicular; that is, the ſum of the forces E and F will be to the power P, which ſuſtains them, as the ſum of the ſides of the wedge to the baſe, or as one ſide to half of the baſe; that is, as the radius to the ſine of half the vertical angle of the wedge. Hence, when in cleaving timber the wedge fills the cleft, in which caſe the reſiſtance of the wood acts perpendicularly on the ſides of the wedge, the power which drives the wedge muſt be to the coheſive force of the timber in a proportion ſomewhat greater than that above mentioned, in order that it may divide the timber, whoſe parts will then recede in lines perpendicular to the ſides of the wedge.

CASE II.

Let now the reſiſting forces of E and F be ſuppoſed to act obliquely on the ſides of the wedge in the directions E K and F L, and let theſe forces be expreſſed by the lines E K and F L, and let each of them be reſolved into two forces, expreſſed reſpectively by the lines E G, G H, and F H, H L, whereof the forces G H and H L, by acting parallel to the ſides of the wedge, are loſt: while the other forces, E G and F H, by acting perpendicularly on the ſides of the wedge, keep the power P in æquilibrio; therefore by the firſt caſe theſe parts of the whole reſiſting force are to the power P, as radius to the ſine of half the vertical angle of the wedge. But it is evident that the whole reſiſting force is to its parts expreſſed by E G, F H, as radius to the ſine of the angle E K G, or F L H; and therefore (compounding theſe ratios) the whole reſiſting force will be to the power which

ſuſtains

ſuſtains it, as the ſquare of radius, to a rectangle under the ſine of the angle which the directions of the reſiſting force make with the ſides of the wedge, and the ſine of half the vertical angle of the wedge. Now, ſince the force of the wedge is exerted in lines perpendicular to the ſurface of its ſides, theſe reſiſting bodies will naturally recede in that direction; as we ſuppoſe them, in this caſe, free to move in any direction whatever.

CASE III.

Let us ſuppoſe laſtly, that the reſiſting bodies are confined, by planes put under them, to recede in the directions K E, L F, then the power which drives the wedge and the reſiſting force will be in æquilibrio, when the former is to the latter, as the ſine of half the vertical angle of the wedge, to the ſine of the angle E K G, F L H, that each ſide of the wedge makes with the direction in which the reſiſting force is confined to recede. For in the firſt caſe it was proved that the power P, which drives the wedge, is to the force with which it protrudes bodies in directions perpendicular to the ſides, as the ſine of half the vertical angle of the wedge to radius. Let then the line G E, which is perpendicular to the ſide A B, expreſs the force with which the power P protrudes the reſiſting bodies in the directions G E, and H F, and let this force be reſolved into two forces, expreſſed by the lines G O, and O E, one perpendicular and the other parallel to K E, the direction in which the reſiſting bodies are confined to move; then the force G O is loſt, and only O E has effect in protruding the

 reſiſting

refifting bodies in the directions K E, and L F. This force therefore being to the force expreffed by G E, as the fine of the angle (E G O, or) E K G, to the radius, and the Force G E being (as was faid before) to the power P, as the radius to the fine of half the vertical angle of the wedge; it follows, that the force with which the refifting bodies are protruded in the directions K E, and L F, is to the power P, as the fine of the angle E K G, or F L H, which thefe directions make with the fides of the wedge, to the fine of half the vertical angle of the wedge: and confequently, if the refifting forces, which act on the wedge according to thefe directions, are to the power P in this proportion, there will be an æquilibrium between them.

Hence we may obferve, that, if from D (the middle point in the back of the wedge) a line be drawn, as D A, meeting one of the fides; the refifting forces, which muft recede in directions parallel to D A; will be to the power which fuftains them, as D B, the height of the wedge, to the line D A; and this power, if at all increafed, will remove thefe refifting bodies. When therefore the refifting bodies muft recede in lines parallel to the back of the wedge, their refiftance will be to the power which fuftains it, as the height of the wedge, to half the breadth of it's back. This proportion of the power to the refiftance in this laft mentioned cafe, is confirmed by an experiment ufed by Gravefande and others, to fhew the nature of the wedge. For, in this experiment, a wedge is drawn down between two cylinders, which roll on rulers parallel to the back of the wedge, and are kept together by weights. And probably it was from their attending to this experiment, without confidering other cafes, that

that they concluded the ſame proportion between the power and reſiſtance would obtain in general.

I have already mentioned the proportion which the power that drives the wedge muſt have to the reſiſtance in cleaving timber, when the wedge exactly fills the cleft; which caſe however ſeldom happens; for the wood generally ſplits to ſome diſtance before the wedge. And then, in order that there may be an æquilibrium between the power driving the wedge and the reſiſtance of wood, the former muſt be to the latter, as the ſine of half the vertical angle of the wedge, to the coſine of the angle which the ſide of the cleft makes with the ſide of the wedge. The truth of which is eaſily underſtood from what was proved in the third caſe of the wedge; for the coſine of the angle, contained between the ſide of the cleft and the ſide of the wedge, is the ſine of the angle which the ſide of the wedge contains with the direction in which the wood recedes; becauſe, as the cleft opens, the wood muſt recede in lines perpendicular to the ſides of the cleft; and in the direction of thoſe lines doth the reſiſtance of the wood act on the ſides of the wedge.

The inclined plane is reckoned by ſome writers among the mechanic powers; and I think with reaſon, as it may be uſed with advantage in raiſing weights.

Let the Line A B [TAB. VI. Fig. 4.] repreſent the length of an inclined plane, AD its height, and the line BD we may call its baſe. Let the circular body GEF, be ſuppoſed to reſt on the inclined plane, and to be kept from falling down it by a ſtring C S tyed to its center C. Then the force with which this body ſtretches the ſtring will be to its whole weight, as the ſine of A B D, the angle of elevation, to the ſine of the angle which the ſtring contains with a line perpendicular to A B the length of the plane. For let the radius C E be drawn perpendicular to the horizon, and C F perpendicular to A B: and from E draw E O parallel to the ſtring and meeting C F in O. Then, as the body continues at reſt and is urged by three forces, to wit, by its weight in the direction C E, by the reaction of the plane in the direction F C, and by the reaction of the ſtring in the direction E O; the reaction of the ſtring, or the force by which it is ſtretched, is to the weight of the body, as E O to C E: that is, as the ſine of (the angle C E F, which is equal to) A B D, the angle of elevation, to the ſine of the angle E O C, equal to S C O, the angle which the ſtring contains with the line C F perpendicular to A B, the length of the plane.

When therefore the ſtring is parallel to the length of the plane, the force with which it is ſtretched, or with which the body tends down the inclined plane, is to its whole weight, as the ſine of the angle of elevation, to the radius, or as the height of the plane to the length. And in the ſame manner it may be ſhewn that, when the ſtring is parallel to B D, the baſe of the plane, the force with which it is ſtretched is to the weight of the body, as A D to B D, that is, as the height of plane to its baſe. If we ſuppoſe the ſtring, which ſupports the body G E F, to be faſtened at S, and that a force, by acting on the line A D, the height of the plane, in a direction parallel to the baſe B D, drives the inclined plane under the body, and by that means makes it riſe in a direction parallel to A D.

Then

Then, from what was proved in the third caſe of the wedge, it will appear, that this force muſt be to the weight of the body, as A D to B D, or rather in a proportion ſomewhat greater: if it makes the plane move on and the body riſe.

From this laſt obſervation we may clearly ſhew the nature and force of the ſcrew; a machine of great efficacy in raiſing weights or in preſſing bodies cloſely together. For if the triangle A B D be turned round a cylinder whoſe periphery is equal to B D, then the length of the inclined plane B A will riſe round the cylinder in a ſpiral manner, and form what is called the thread of the ſcrew: and we may ſuppoſe it continued in the ſame manner round the cylinder from one end to the other; and A D the height of the inclined plane will be every where the diſtance between two contiguous threads of this ſcrew, which is called a convex ſcrew. And a concave ſcrew may be formed to fit this exactly, if an inclined plane every way like the former be turned round the inſide of a hollow cylinder, whoſe periphery is ſomewhat larger than that of the other. Let us now ſuppoſe the concave ſcrew to be fixed, and the convex one to be fitted into it, and a weight to be laid on the top of the convex ſcrew: Then, if a power be applied to the periphery of this convex ſcrew to turn it round, at every revolution the weight will be raiſed up thro' a ſpace equal to the diſtance between the two contiguous threads, that is to the line A D the height of the inclined plane B A; therefore ſince this power, applied to the periphery, acts in a direction parallel to B D, it muſt be to the weight it raiſes as A D to B D, or as the diſtance between two contiguous threads, to the periphery of the convex ſcrew.

N. B.

N. B. The diſtance betwen two contiguous threads is to be meaſured by a line parallel to the axle; if we now ſuppoſe that a hand-ſpike or handle is inſerted into the bottom of the convex ſcrew, and that the power which turns the ſcrew is applyed to the extremity of this handle, which is generally the caſe; then as the power is removed farther from the axis of motion, its force will be ſo much encreaſed (vide what was ſaid of the lever, cor 1.) and therefore ſo much may the power itſelf be diminiſhed. So that the power, which, acting on the end of a handle, ſuſtains a weight by means of a ſcrew, will be to that weight, as the diſtance between two contiguous threads of the ſcrew, to the periphery deſeribed by the end of the handle. In this caſe we may conſider the machine as compoſed of a ſcrew and a lever, or, as Sir Iſaac Newton expreſſeth it, *Cuneus a vecte impulſus.*

I have now given you my ſentiments, as to the principles on which, I think, the efficacy of the mechanic powers may be moſt properly explained; and hope that, where I have preſumed to differ from others, you will think I have ſome appearance of reaſon on my ſide. I find my paper has been drawn out much beyond what I at firſt expected, and I fear much beyond your patience; and therefore ſhall detain you no longer than to aſſure you that I am, Sir,

with the ſincereſt regard, your

moſt obedient humble ſervant,

Hugh Hamilton.

XXVI. *An*

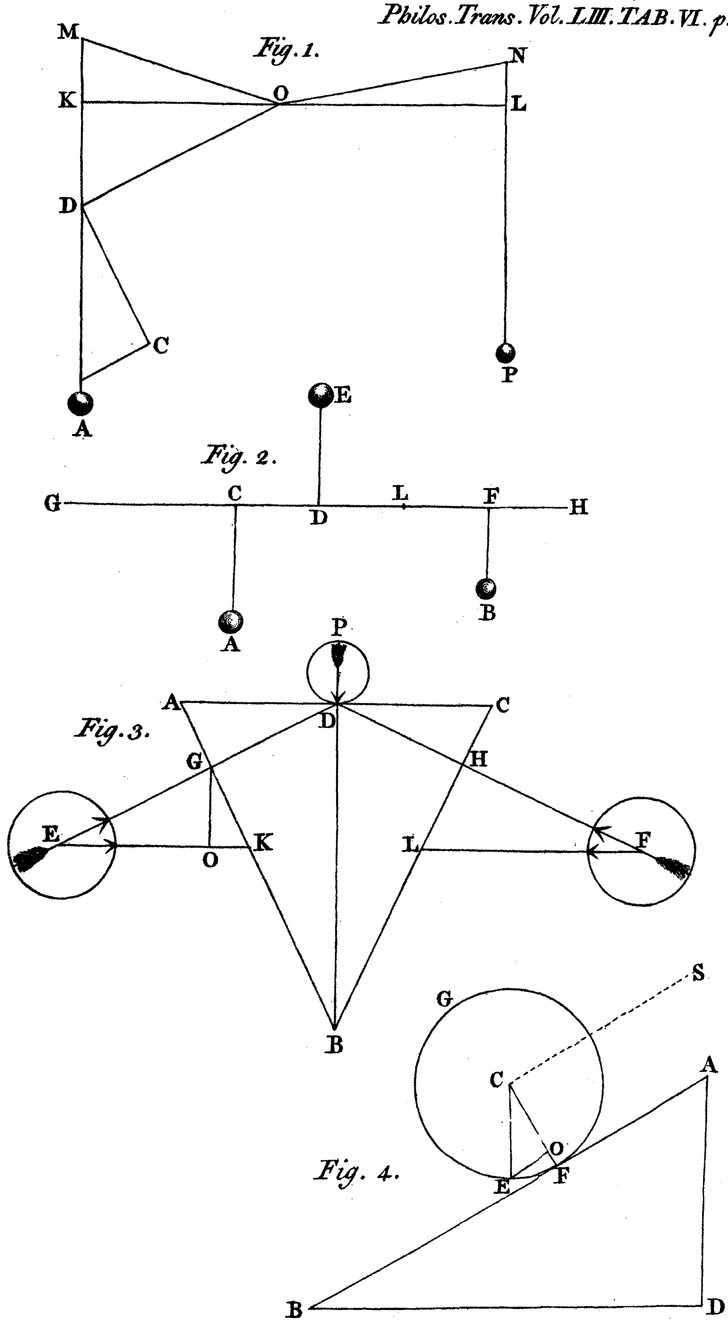
Fig. 1.
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Fig. 2.
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Fig. 3.
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Fig. 4.
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J. Mynde sc.